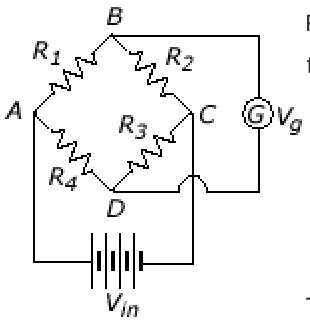
Basic Wheatstone Bridge Circuit

A basic Wheatstone bridge circuit contains four resistances, a constant voltage input, and a voltage gage, as illustrated below.



For a given voltage input V_{in}, the currents flowing through *ABC* and *ADC*depend on the resistances, i.e.,

$$V_{\mathrm{in}} = V_{ABC} = V_{ADC}$$
$$\Rightarrow I_{ABC}(R_1 + R_2) = I_{ADC}(R_4 + R_3)$$

The voltage drops from *A* to *B* and from *A* to *D* are given by,

$$\begin{cases} V_{AB} = I_{ABC}R_1 = \dfrac{V_{\mathrm{in}}}{R_1 + R_2} R_1 \\ V_{AD} = I_{ADC}R_4 = \dfrac{V_{\mathrm{in}}}{R_4 + R_3} R_4 \end{cases}$$

The voltage gage reading V_g can then be obtained from,

$$V_g = V_{AB} - V_{AD} = \frac{V_{\mathrm{in}}}{R_1 + R_2} R_1 - \frac{V_{\mathrm{in}}}{R_4 + R_3} R_4$$
$$= \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_4 + R_3)} V_{\mathrm{in}}$$

Now suppose that all resistances can change during the measurement. The corresponding change in voltage reading will be,

$$V_g + \Delta V_g = \frac{(R_1 + \Delta R_1)(R_3 + \Delta R_3) - (R_2 + \Delta R_2)(R_4 + \Delta R_4)}{(R_1 + \Delta R_1 + R_2 + \Delta R_2)(R_4 + \Delta R_4 + R_3 + \Delta R_3)} V_{\mathrm{in}}$$

Balanced Wheatstone Bridge Circuit

If the bridge is **initially balanced**, the initial voltage reading V_g should be zero. This yields the following relationship between the four resistances,

$$V_g = \frac{R_1R_3 - R_2R_4}{(R_1+R_2)(R_4+R_3)} V_{in} = 0$$

$$\Rightarrow R_1R_3 = R_2R_4 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{R_4}{R_3} = \frac{1}{r}$$

We can use this result to simplify the previous equation that includes the changes in the resistances. Doing so results in the solution for the change in V_g,

$$\Delta V_g = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right)(1+\eta)V_{in}$$

where η is defined by,

$$\eta = \frac{1}{1 + \cfrac{1+r}{\cfrac{\Delta R_1}{R_1} + \cfrac{\Delta R_4}{R_4} + r\left(\cfrac{\Delta R_2}{R_2} + \cfrac{\Delta R_3}{R_3}\right)}}$$

Moreover, when the resistance changes are small (< 5%), the second order term η is approximately zero and can be ignored. We then have,

$$\Delta V_g \approx \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right)V_{in}$$

which is the basic equation governing the Wheatstone bridge voltage in strain measurement.

The coefficient $\frac{r}{(1+r)^2}$ is called the **circuit efficiency**.

Equal-Resistance Wheatstone Bridge Circuit

In practice, one often uses the same resistance value for all four resistors, $R_1 = R_2 = R_3 = R_4 = R$. Noting that $r = 1$ in this case, the change in voltage can be further simplified to,

$$\Delta V_g \approx \frac{\Delta R_1 - \Delta R_2 + \Delta R_3 - \Delta R_4}{4R} V_{in}$$

By thoughtfully selecting the target and reference resistances, the Wheatstone bridge circuit can amplify small changes in resistance and/or compensate for changes in temperature.